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MEASUREMENT OF SOME HYDRODYNAMIC CHARACTERISTICS IN A MECHANICALLY MIXED LIQUID

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The paper lists assumptions permiting the use of the method of three directional probes for measurement of local, time-averaged values of the velocity vector and the static pressure in a mixed system. A concrete application of the method is proposed for measurement of these quantities in the proximity of the bottom of a vessel with radial baffles. The results of the measurement are compared with the values determined by another independent method. The spatial distribution of the measured quantities and their dependence on the frequency of revolution of the mixer agree well with the results measured earlier.

Mixing of liquids displays from hydrodynamical point of view a rather complicated pattern of the flow. It is affected primarily by geometrical arrangement, *i.e.* the shape of mechanical mixer, the shape of vessel, eventually by different types of baffles.

A more profound knowledge of this process aimed at expedient design and operation of mixing equipment requests therefore a more detailed knowledge of some hydrodynamic characteristics, and, in particular, local values of velocities and/or pressure. Intensity of turbulence in the equipment is an important factor necessitating that these quantities be taken into consideration. The knowledge of their time-averaged values provides valuable and, as a rule, also sufficient information for given purposes.

Experimental methods enabling the measurement of these characteristics are described in length in the literature. Their evaluation from the standpoint of their use in a mechanically mixed charge has been submitted by one of the authors elsewhere[1]. It has been shown there too, that the pressure probe, namely the "three-opening Pitot tube" designed by Jezdinský[2], is a suitable device for determination of local, time-averaged quantities in a mixing apparatus.

In this communication we shall give a modification of the three-opening Pitot tube method for measurements in the vicinity of walls or bottom of a mixed vessel. The velocity field here markedly affects the rate of heat transfer between the jacket of the vessel and the mixed batch. The proposed method can be used for measurement

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of planar or axially symmetric flow, *i.e.* in equipment where this assumption is at least approximately fulfilled. A more sophisticated probe has to be used for other types of flow.

THEORETICAL

A somewhat modified method due to Godstein[3] will be used here to derive relationships for the calculations. First, we write the Reynolds equation of a stationary turbulent flow of a viscid incompressible fluid, making following assumptions: *1*. Viscous stresses may be neglected in comparison with the turbulent stresses. *2*. The turbulent field is locally isotropic. *3*. Body forces are conservative. The Reynolds equation takes then following form

$$\bar{\mathbf{w}} \cdot \operatorname{grad} \bar{\mathbf{w}} = - \operatorname{grad} \pi + \operatorname{div} \bar{\mathbf{T}} \,, \tag{1}$$

where π is the potential of the body force and the bar over the letter denotes corresponding time-averaged quantities. The matrix of the tensor $\bar{\mathbf{T}}$ is given by

$$\bar{t}_{ij} = - \bar{p}_{ij} - \varrho(\overline{w'^2}/3)\,\delta_{ij} \,,$$

$$\text{where} \quad \delta_{ij} \begin{cases} = 0\,[i \neq j] \\ = 1\,[i = j] \,. \end{cases}$$

The vector field of average velocities $\bar{\mathbf{w}}$ determines a set of permanent streamlines. Eq. (*1*) can be integrated along a streamline with the following results

$$[\varrho(\bar{w}^2/2) + \varrho(\overline{w'^2}/3) + \pi + p]\,|_{r_1} = [(\bar{w}^2/2) + (\overline{w'^2}/3) + \pi + p\,|_{r_2} \,; \tag{2}$$

$\mathbf{r}_1$ and $\mathbf{r}_2$ designate position vectors assigned to two points on a permanent streamline.

Let us assume now that in the point given by $\mathbf{r}_2$ the liquid hits a stagnant surface of inifinitesimal magnitude dA, forming a base of a solid, obliquely cut cylinder S (Fig. 1). β is then the angle between the vector $\bar{\mathbf{w}}$ and the normal, $\mathbf{n}$, to this surface. In order that the behaviour of the liquid can be described, we make two additional assumptions: *4*. The deceleration of liquid in the point $\mathbf{r}_2$ is such that the diminishing of the vector of the average velocity is an unambiguous function of mutual orientation of the velocity vector and the surface dA. *5*. The distance between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is such that the presence of the solid body does not affect the flow field in the point $\mathbf{r}_1$, but the difference of potentials in these points due the body force is negligible owing to the pressure changes.

In accord with assumption *4* we have that

$$|\overline{\mathbf{w}(\mathbf{r}_2)}| = |\overline{\mathbf{w}(\mathbf{r}_1)}|\,\varphi(\beta)\,, \quad [\beta < \pi/2] \,.$$

The normal component of the stress tensor $\overline{\boldsymbol{T}}$ on the surface dA is given as a sum of the static and the turbulent stresses

$$\overline{t_n(\boldsymbol{r}_2)} = \overline{p(\boldsymbol{r}_2)} + \varrho[\overline{w'^2(\boldsymbol{r}_2)}]/3 .$$

Substituting these relations into Eq. (2) and making use of assumption (5) we obtain

$$\overline{t_n(\boldsymbol{r}_2)} = \varrho(\overline{w(\boldsymbol{r}_1)/2})^2 [1 - \varphi^2(\beta)] + \varrho(\overline{w'^2(\boldsymbol{r}_1)/3}) + \overline{p(\boldsymbol{r}_1)} .$$

For a liquid at rest the last equation simplifies to

$$t_n(\boldsymbol{r}_2) = p(r_1) = p_0(r_2) .$$

Substracting the last two relations we obtain finally expression

$$\overline{\Delta p} = \overline{\Delta p_{dyn}} + \overline{\Delta p_{st}} , \tag{3}$$

where the quantity

$$\overline{\Delta p_{dyn}} \equiv \varrho[\overline{w(\boldsymbol{r}_1)^2/2}]\, \psi[\beta(\boldsymbol{r}_1)] ,$$

$$\psi[\beta(\boldsymbol{r}_1)] \equiv 1 - \varphi^2[\beta(\boldsymbol{r}_1)] ; \quad \overline{w(\boldsymbol{r}_1)} \equiv |\overline{w(\boldsymbol{r}_1)}| \tag{3a}$$

is the dynamic pressure, the quantity

$$\overline{\Delta p_{st}} \equiv \varrho[\overline{w'^2(\boldsymbol{r}_1)/3}] + \overline{p(\boldsymbol{r}_1)} - p_0(\boldsymbol{r}_2) \tag{3b}$$

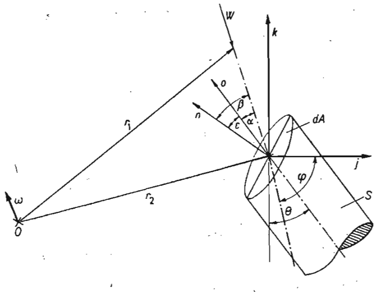

FIG. 1
Orientation of the Probe and the Vector of Average Velocity $\overline{\boldsymbol{w}}$ in the Plane M

is the static pressure and the quantity

$$\overline{\Delta p} \equiv \overline{t_n(\mathbf{r}_2)} - p_0(\mathbf{r}_2) \qquad (3c)$$

is the total pressure.

Consider now that a measuring probe shaped as an obliquely cut cylider with sufficiently small base is available with the possibility to measure the pressure exerted by the liquid on this surface. Further, let us have a calibrating device simulating a field in which both the direction and the magnitude of velocity, as well as the static pressure are known functions of position. We shall show that in such a case these quantities can also be determined in an arbitrary point in the mixed batch, provided that the above assumptions are fulfilled.

Consider a plane M containing the vector of the average velocity $\overline{\mathbf{w}}$ (Fig. 1). In this plane we select an orthonormal base of vectors $\mathbf{j}$, $\mathbf{k}$. The cylinder S is oriented with its axis (given by the vector $\mathbf{o}$) and the normal, $\mathbf{n}$, of its base, $\mathrm{d}A$, being in the same plane. From Fig. 1 it is seen that the angle φ determines the orientation of the vector $\overline{\mathbf{w}}$ and the angle θ orientation of the vector $\mathbf{o}$ with respect to the base. Mutual position of the cylinder S and the average velocity vector is given by the angles α and β; the geometry of the cylinder by the angle ε.

For these angles we have following relations

$$\beta = \alpha + \varepsilon\,, \qquad (4a)$$

$$\varphi = \pi/2 - \theta + \chi\,. \qquad (4b)$$

In a mixed system, we determine the triples of quantities $\{\overline{w}, \varphi, \overline{\Delta p_{st}}\}$ as functions of position. It is apparent that a suitably chosen sequence of finite changes of the geometry or orientation of the probe (*i.e.* the angles θ or ε) will enable us to obtain necessary set of experimental data of $\overline{\Delta p}$ and to form necessary number of relations for evaluating the sought quantities. The procedure is as follows. The values of the angles $\{\varepsilon_i\}$ $(i = 1, 2, 3)$ are selected. Then on the basis of Eqs (3) and (3a) three equations for a given position in the system are set up.

$$\overline{\Delta p_i} = \overline{\Delta p_{idyn}} + \overline{\Delta p_{st}} = \Phi(\overline{w})\,\psi_i(\alpha + \varepsilon_i) + \overline{\Delta p_{st}}\,,$$

$$\Phi(\overline{w}) \equiv \varrho(\overline{w}^2/2)\,, \quad (i = 1, 2, 3)\,. \qquad (5)$$

We shall show now that these relations, together with Eq. (4b), enable one to solve the given problem provided that the function $\psi_i(\alpha + \varepsilon_i)$ may be found independently. First, to eliminate the quantity $\overline{\Delta p_{st}}$, multiply each of the equations by a so far undetermined coefficients x_{i1} and substract the obtained relations. The whole procedure is repeated once again with coefficients x_{i2}.

We put $\sum_{i=1}^{3} x_{ij} = 0\,, \quad (j = 1, 2)$

to obtain two independent relations which do not contain $\overline{\Delta p_{st}}$

$$\sum_{i=1}^{3} x_{ij}\,\overline{\Delta p_i} = \sum_{i=1}^{3} x_{ij}\,\Phi(\overline{w})\,\psi_i(\alpha + \varepsilon_i)\,, \quad (j = 1, 2)\,. \tag{5a}$$

Further, we eliminate $\Phi(\overline{w})$ by dividing one of the relations (*5a*) by other

$$\frac{\sum_{i=1}^{3} x_{i1}\,\overline{\Delta p_i}}{\sum_{i=1}^{3} x_{i2}\,\overline{\Delta p_i}} = \frac{\sum_{i=1}^{3} x_{i1}\,\varphi_i(\alpha + \varepsilon_i)}{\sum_{i=1}^{3} x_{i2}\,\varphi_i(\alpha + \varepsilon_i)} \equiv \mathrm{f}(\alpha)\,. \tag{6}$$

It is apparent that for fixed values of ε_i the last relation is a function of the angle α only.Thus for properly selected values x_{ij} the angle α can be determined unambiguously on the basis of the experimental values of $\overline{\Delta p_i}$.

To determine the static pressure we divide one of Eqs (*5a*) by one of Eqs (*5*)

$$\frac{\sum_{i=1}^{3} x_{ij}\,\overline{\Delta p_i}}{\overline{\Delta p_i} - \overline{\Delta p_{st}}} = \frac{\sum_{i=1}^{3} x_{ij}\,\psi_i(\alpha + \varepsilon_i)}{\psi_i(\alpha + \varepsilon_i)} \equiv \mathrm{F}(\alpha)\,. \tag{6a}$$

This relation is a function of the already determined agle α too, so that on substituting from Eq. (*6*) into (*6a*) and some rearrangement we obtain an explicit relationship for $\overline{\Delta p_{st}}$

$$\overline{\Delta p_{st}} = \overline{\Delta p_i} - \sum_{i=1}^{3} x_{ij}\,\overline{\Delta p_i}/\mathrm{F}\left[f^{-1}\left(\frac{\sum_{i=1}^{3} x_{i1}\,\overline{\Delta p_i}}{\sum_{i=1}^{3} x_{12}\,\overline{\Delta p_i}}\right)\right].$$

Now it is easy to determine the value of $\Phi(\overline{w})$, resp. $\overline{w}$, from Eq. (*5*)

$$\Phi(\overline{w}) = \frac{\overline{\Delta p_i} - \overline{\Delta p_{st}}}{\psi_i(\chi + \varepsilon_i)}\,.$$

The coefficients x_{ij} are still to be selected. The choice is most convenient when the form of the function $\mathrm{f}(\alpha)$ and $\mathrm{F}(\alpha)$ permits interpolation of functional values or the argument α with sufficient accuracy. Therefore we request that in the range $|\alpha| < \pi/6$ these functions are *a*) continuous, *b*) monotone, *c*) even, *i.e.* symmetric with respect to the value $\alpha = 0$.

The requirement of symmetry makes interpolation easier, however, it is not necessary. It is associated primarily with the possibility of precise calibration and construction of the elements of probe. As will be shown below (Fig. 2) the probes are symmetric about the axis passing through the middle one. In a case when requirement *c*) is satisfied but the form of functions $f(\alpha)$ and $F(\alpha)$. determined by calibration is not symmetric, it may be inferred that a systematic error occurred. The symmetry of the geometric arrangement of the measuring elements is achieved by choosing the values of the angles ε_i as follows

$$\varepsilon_1 = \gamma ; \quad \varepsilon_2 = 0 ; \quad \varepsilon_3 = -\gamma .$$

It may be shown then that the values x_{ij} are determined with the accuracy of an arbitrary non-zero factor. As follows from Eq. (*3a*), each of the functions $\psi_i(\beta_i)$ is even with respect to corresponding β_i, while on the basis of the model of the momentum loss in liquid (see assumption No 4) it is apparent that $\psi_i(0) \neq 0$. Let us choose now the value of the constants x_{1j}. The function $x_{1j}[\psi_1(\alpha + \gamma) - \psi_3(\alpha - \gamma)]$ is then odd and the function $x_{1j}[\psi_1(\alpha + \gamma) - 2\psi_2(\alpha) + \psi_3(\alpha - \gamma)]$ is even with respect to the argument α. In order that the function $f(\alpha)$ and $F(\alpha)$ may satisfy given requirements, and considering Eqs (*5*) and (*6*), they must have the following form

$$f(\alpha) = \frac{\overline{\Delta p_1} - \overline{\Delta p_3}}{\overline{\Delta p_1} - 2\,\overline{\Delta p_2} + \overline{\Delta p_3}}\, B ;$$

$$F(\alpha) = \frac{\overline{\Delta p_{1\mathrm{dyn}}} - \overline{\Delta p_{3\mathrm{dyn}}}}{\overline{\Delta p_{2\mathrm{dyn}}}}\, C . \qquad (6b)$$

In our experiments we put $B = -2$ and $C = 1$.

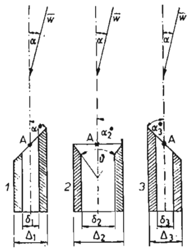

FIG. 2
Sketch of the Directional Probes

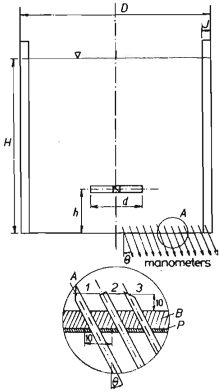

FIG. 3
Scheme of Experimental Set-up
B Bottom of vessel, *P* supporting slab.

Since $\psi_i(\alpha + \varepsilon_i)$ is not an explicit function of position, $f(\alpha)$ and $F(\alpha)$ must take the same values both in the calibration device and in the mixed charge if the angles ε_i are preserved. In the calibration device the form of these functions can be found (eventually the form of $\Phi(\overline{w})$ can be verified) so that the values $\{\overline{w}, \varphi, \overline{\Delta p_{st}}\}$ may be determined on the basis of three measured values of $\overline{\Delta p_i}$:

$$\varphi = \pi/2 - \theta + f^{-1}(\alpha) ; \tag{7}$$

$$\overline{\Delta p_{st}} = \overline{\Delta p_2} - (\overline{\Delta p_1} - \overline{\Delta p_3})(1/F(\alpha)) ; \tag{8}$$

$$\overline{w} = \sqrt{\left(\frac{2}{\varrho} \frac{\overline{\Delta p_i} - \overline{\Delta p_{st}}}{\psi_i(\alpha + \varepsilon_i)}\right)} . \tag{9}$$

As will be shown in the following index i in the last equation is chosen so as to keep the value $|\chi - \beta_i|$ minimum.

EXPERIMENTAL

Description of the directional probes. In the construction of the probes we started from the requirements listed in the preceding paragraph. We used three types of probes (see Fig. 2) with the dimensions (inner diameter δ_i, outer diameter Δ_i and the bevel angle α_i^*) summarized in Table I. It is apparent that the angle α_i^* is a complementary of ε_i defined in Theoretical. The opening of the probe number 2 was tapered, the apex angle is designated in Fig. 2 as ϑ. This arrangement, though enlarging the opening, permits measurement of the average velocity in a wider range of the angle β regardless of the orientation of the probe. The probes were constructed from hypodermic needles of the length about 80 mm. Each probe was connected to a manometer from which the pressure differences were visually read-off. Sufficient time was allowed in experiments for the level in the manometric tubes to steady down until three consequent readings of the pressure differred by not more than the accuracy of the manometer, *i.e.* $\pm 0{\cdot}5$ mm of water head. At 30° inclination of the manometer this corresponds to about $\pm 2{\cdot}5\ N/m^2$.

Calibration of the probes. The probes were calibrated in a cylinder filled with liquid and rotating at a constant angular velocity, ω, about its vertical axis, *i.e.* in the equipment described earlier[1] in detail. The orientation of the probes in the calibrating device can be characterized,

TABLE I

Characteristics of the Directional Probes

Number of probe	δ_i mm	Δ_i mm	α_i^* deg	ϑ deg	K_i —	σ_{K_i} —	β_i' deg	β_i'' deg
1	0·4	0·8	42·5	—	0·935	0·009	−20	—
2	0·6	1·0	90	60	0·900	0·014	−10	+10
3	0·4	0·8	−42·5	—	0·935	0·009	—	+20

referring to Fig. 1, as follows:

$$j = \frac{\omega}{|\omega|},$$

$$k = -\left(\frac{\omega}{|\omega|}\right) \times \left(\frac{r_2}{|r_2|}\right) = -\frac{w}{|\omega|\,|r|}.$$

The origin of the coordinate system is placed on the axis of the rotating cylinder. The plane M is therefore a tangent plane to the cylindrical surface, the angle φ equals 90° and consequently $\alpha = \theta$. The orientation of the probe could be set in the range of the angle $\theta \langle -50°, +50° \rangle$ with the accuracy $\pm 1°$.

Measurement in mixed system. The experiments were carried out in a cylindrical perspex vessel $D = 290$ mm in inner diameter. The vessel was equipped with four to the bottom reaching radial baffles of the width J, equal one tenth of D (Fig. 3). The charge was water kept at 20°C temperature with the accuracy ± 1°C by a thermostat. The depth of liquid in the vessel, H, equalled the diameter of the vessel. A six-paddle mixer with flat blades inclined by 45° [1,8] and the diameter, d, equalling one third, respectively one fourth of the diameter of the vessel was used and rotated so as to drive liquid toward bottom. The stirrer was driven by a 0·4 kW DC electric motor and the frequency of revolution was controlled by means of a Rome regulator with the accuracy $\pm 1\%$ in the range 300–2500 r.p.m. The frequency of revolution was measured by a photoelectric tachometer[4] with the accuracy of one revolution per measured time interval. The measurements were carried out near the bottom in a vertical plane of symmetry between two neighbouring baffles. This plane is therefore identical with M, the orientation of which was expressed in our experimental set-up by relations

$$k = \frac{\omega_m}{|\omega_m|},$$

$$j = \left(\frac{\omega_m}{|\omega_m|}\right) \times \left[\left(\frac{\omega_m}{|\omega_m|}\right) \times \left(\frac{r_2}{|r_2|}\right)\right].$$

The origin of the coordinate system is placed into the point where the axis of the vessel intersects the bottom.

The probes were mounted in the openings in the bottom 10 mm apart. The number of probes located on a radius in the bottom totaled 13. The height of centers of the openings over the bottom was 10 mm with the accuracy $\pm 0{\cdot}2$ mm. Position vectors assigned to individual measuring spots had following cylindrical coordinates:

$$r_{2i} = r_{2i}(ri, \phi, z)\,, \quad (i = 1, 2, \ldots, 13)\,; \quad \phi = \text{const}\,; \quad z = \text{const}.$$

The angle θ in one experiment was the same for all probes and generally took values 0°, 20°, 40°, 60°. The measurements were organized so that a certain type of probe was used under given conditions and replaced by another type of probe under otherwise the same conditions in the next run. (The detail of Fig. 3 shows these types simultaneously). The value of the static pressure calculated on the basis of the data measured by three directional probes according to Eq. (*8*) was checked by direct measurement using a so-called static probe[1,5]. It is a tube sealed on one end and connected to a liquid manometer on the other end. At a certain distance from the sealed end there are opening spaced symmetrically with respect to the axis of the tube. The measurements with that device were performed so that first the orientation of the vector of average velocity was found by means of the directional probes and the static probe was set accordingly.

RESULTS AND DISCUSSION

The form of the functions f(α) and F(α) defined in Eq. (6) was found on the calibrating device. The plot of the function f(α) is shown in Fig. 4. The function F(α) has a similar course[6]. In the range of the angle α which does not deviate appreciably from zero both functions were approximated by a straight line

$$\eta_r = m_r\alpha + q_r \quad [\alpha'_r < \alpha < \alpha''_r]\,, \quad r = \begin{cases} \mathrm{f} \\ \mathrm{F} \end{cases}. \tag{10}$$

The values m_r and q_r and their standard deviations determined by the least square method are summarized in Table II showing also the validity limits for functions η_r.

The form of functions $\psi_i(\beta_i)$ was searched on the same device. Since the openings of the probes have finite dimensions in reality (the opening of the probe 2 is tapered), the functions exhibit flat maxima. In a certain range of the angles β_i the value of these functions may be regarded as constant; this quantity is referred to sometimes as the resistance coefficient of the probe K_i

$$\psi_i(\beta_i) \equiv K_i = \text{const.}\,, \quad [\beta'_i < \beta_i < \beta''_i\,.]\,.$$

The values of the coefficients K_i, their standard deviations and the values of the angles β'_i and β''_i delimiting the validity are summarized in Table I.

The values of the total pressure $\overline{\Delta p}$, as functions of the radial coordinate r, were measured in the mixed batch 10 mm above the bottom. Then, on the basis of the function determined by calibration, the values of the angle φ, the velocity $\overline{w}$ and the static pressure $\overline{\Delta p_{st}}$ were calculated from Eqs (7)–(9) as functions of r. An example of the $\varphi = \varphi(r)$ dependence is shown in Fig. 5. The determined values of the velocity (or its

TABLE II

Parameters of the Calibration Functions

Subscript r	m_r deg^{-1}	q_r —	σ_{m_r} deg^{-1}	σ_{q_r} —	α'_r deg	α''_r deg
f	0·073	0·232	0·003	0·049	−30	+30
F	0·042	0·123	0·001	0·022	−30	+30

radial and axial components) and the values of the static pressure were expressed as dimensionless quantities by relating them to the velocity of the tips of stirrer blades:

$$W_{ax} \equiv \bar{w} \sin \varphi / \pi d n \,,$$
$$W_{rad} \equiv \bar{w} \cos \varphi / \pi d n \,,$$
$$\Delta P_{st} \equiv 2 \Delta p_{st} / [\varrho (\pi d n)^2] \,. \qquad (11)$$

Dependence of thus defined quantities on the position vector $\mathbf{r}$ is shown in Figs 6 and 7. Solid lines in Figs 5 through 7 give correlations, the parameters of which were calculated by the least square method. No simple function approximating the dependence $\Delta P_{st} = \Delta P_{st}(r)$ was found.

The check values of the static pressure, measured by the "static probe", were compared statistically with the values calculated on the basis of Eq. (8) by testing statistical equivalence of the expected values[7]. It has been found on 95% significance level that the results are in agreement. On the basis of this finding it may be concluded that the assumptions made in part Theoretical, and particularly the assumption of

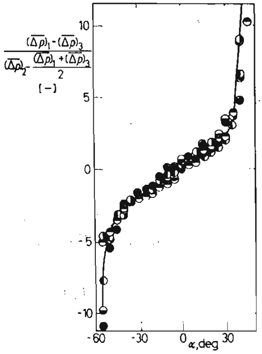

FIG. 4

Course of Function f(α)

w, m/s	0·624	0·700	0·800	0·950	1·070
point	●	◐	◑	◓	◒

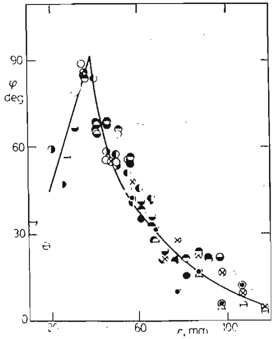

FIG. 5

Dependence of the Angle φ on the Radial Coordinate ($d/D = 1/3$)

n, min^{-1}	350	500	575	350	500
θ, deg	0	0	0	20	20
point	○	◑	◐	●	◓
n, min^{-1}	575	350	500	575	
θ, deg	20	40	40	40	
point	◒	⊙	⊗	⊖	

local isotropic turbulence, are satisfied with sufficient accuracy. Thus the method of three directional probes may be used for the measurement of local flow characteristics in mixed systems not only in regions relatively remoted from the walls of the mixing vessel[1] but also in their vicinity. The values of the variance of individual characteristics measured in the calibration and mixing equipment permit to infer that the error of measurement reaches 15% under unfavourable conditions and in most cases it is less than 10% of the measured value. It is emphasized once again that the experimental arrangement of the probes enables examinations of the flow to be made in one plane. In our case, for instance, the tangential component of velocity could not be determined. It may be speculated, however, that for stirrers inducing axial flow this component in the proximity of a solid body, *i.e.* the bottom of the vessel, is insignificant.

Suitability of the method is quantitatively confirmed also by the course of the characteristics of the flow on the radial coordinate. The streamlines exiting from the

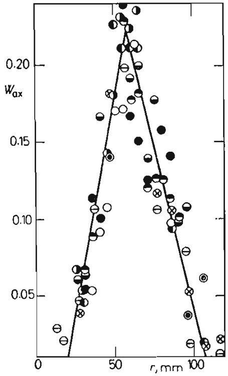

FIG. 6

Dependence of Dimensionless Axial Velocity Component on the Radial Coordinate ($d/D = 1/3$)

n, min^{-1}	350	500	575	350	500
θ, deg	0	0	0	20	20
point	○	◑	◐	●	◓
n, min^{-1}	575	350	500	575	
θ, deg	20	~~20~~ 40	40	40	
point	◒	⊙	⊗	⊖	

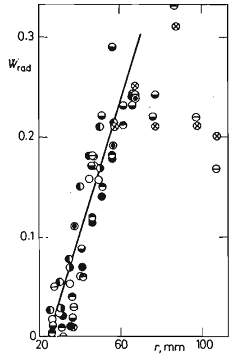

FIG. 7

Dependence of Dimensionless Radial Velocity Component on the Radial Coordinate ($d/D = 1/4$)

n, min^{-1}	600	900	1 100	600	900
θ, deg	0	0	0	20	20
point	○	◑	◐	●	◓
n, min^{-1}	1 100	600	900	1 100	
θ, deg	20	40	40	40	
point	◒	⊙	⊗	⊖	

mixer change their direction is such a manner that the liquid flows toward the wall of the vessel and then turns upwards. Below the mixer there is a region of so-called reversal flow (the angle φ increases here with increasing radial coordinate) where a part of the liquid turns toward the axis of the vessel and forms a vortex there. This is in agreement with the findings published earlier[1,8].

It was further confirmed that under so called automodel regime of the flow the values of the angle φ are independent of the velocity of rotation of the mixer[9-14], the velocity is directly proportional to the frequency of revolution[1,14,15] and values of the static pressure are proportional to the square of the frequency of revolution[1].

LIST OF SYMBOLS

A base of cylinder (m^2)
B coefficient in Eq. (*6*)
C coefficient in Eq. (*6*)
D inner diameter of vessel (m)
d diameter of mixer (m)
$F(\alpha)$ function defined in Eq. (*6b*)
$f(\alpha)$ function defined in Eq. (*6b*)
H depth of liquid at rest (m)
h height of the mixer over bottom (m)
J width of radial baffle (m)
$\mathbf{j}$ unit vector of the base
K coefficient of resistance of the probe
$\mathbf{k}$ unit vector of the base
M plane of flow
m constant in Eq. (*10*) (deg^{-1})
$\mathbf{n}$ unit vector, normal to the base of cylinder S
$\mathbf{o}$ unit vector; direction of axis of the cylinder S
P dimensionless pressure defined by Eq. (*11*)
p pressure ($kg\ m^{-1}\ s^{-2}$)
q constant in Eq. (*10*)
$\mathbf{r}$ position vector (m)
r radial coordinate (m)
S obliquelly cut cylinder
$\mathbf{T}$ stress tensor ($kg\ m^{-1}\ s^{-2}$)
t element of matrix of the stress tensor ($kg\ m^{-1}\ s^{-2}$)
$\bar{u}$ integral time averaged value of u
W dimensionless velocity defined by Eq. (*11*)
$\mathbf{w}$ velocity of liquid ($m\ s^{-1}$)
w' fluctuation component of velocity ($m\ s^{-1}$)
x coefficient
z axial coordinate (m)
α angle between the vector of average velocity and axis of the cylinder S (deg)
α^* bevel angle of the probe (deg)
β angle between the vector of time averaged velocity and the normal to the base of S (deg)
γ angle between axis of cylinder S and normal to its base at symmetric arrangement (deg)

Δ outer diameter of the probe (m)
Δu difference of quantity u
δ inner diameter of the probe (m)
δ Kronecker delta
ε angle between axis of the cylinder S and normal to its base at general arrangement (deg)
$\eta(\alpha)$ function defined by Eq. (*10*)
θ angle between the axis of cylinder S and vector k of the base (deg)
ϑ apex angle of the probe (deg)
π potential of the body force in a unit of volume of liquid ($kg\ m^{-1}\ s^{-2}$)
ϱ density of liquid ($kg\ m^{-3}$)
σ variance
ϕ angular coordinate (deg)
$\phi(\bar{w})$ function defined by Eq. (*5*)
φ angle between the vector of average velocity and vector $\boldsymbol{k}$ of the base (deg)
$\varphi(\beta)$ function characterizing deviation of velocity vector after impact on the stagnant surface
$\psi(\beta)$ function defined by Eq. (*3a*)
ω angular velocity (s^{-1})

Subscripts and Superscripts

ax axial
dyn dynamic
i element of a set
j element of a set
n normal
m mixing system
o at rest
r defined by Eq. (*10*)
rad radial
st static
$'$ lower limit of angles α and β
$''$ upper limit of angles α and β

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